8.3 Graphing Polynomial Functions

Objectives:

8.3a: I can find the multiplicity and end behavior of a polynomial function.

8.3b: I can graph a polynomial function by hand using end behavior, multiplicity, and zeros.

In your note books draw four tables with this information...

Function	Domain	Range	End Behavior	End Behavior
$f(x) = x^2$	$(-\infty, \infty)$	$[0, \infty)$	$x \to -\infty$ $y \to \infty$ ↖	$x \to \infty$ $y \to \infty$ ↗
$f(x) = x^4$			↖	↗
$f(x) = x^6$			↖	↗

Function	Domain	Range	End Behavior	End Behavior
$f(x) = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	$x \to -\infty$ $y \to -\infty$ ↙	$x \to \infty$ $y \to \infty$ ↗
$f(x) = x^3$			↙	↗
$f(x) = x^9$			↙	↗

Function	Domain	Range	End Behavior	End Behavior
$f(x) = -x^2$	$(-\infty, \infty)$	$(-\infty, 0)$	$x \to -\infty$ $y \to -\infty$ ↙	$x \to \infty$ $y \to -\infty$ ↘
$f(x) = -x^4$			↙	↘
$f(x) = -x^6$			↙	↘

Function	Domain	Range	End Behavior	End Behavior
$f(x) = -x$	$(-\infty, \infty)$	$(-\infty, \infty)$	$x \to -\infty$ $y \to \infty$ ↖	$x \to \infty$ $y \to -\infty$ ↘
$f(x) = -x^3$			↖	↘
$f(x) = -x^9$			↖	↘

If you have a graphing calculator graph the functions above to better understand the Domain, Range and end behavior.

We will be graphing these functions on your calculator.

- I'll walk you through the first one...

- Using the graph you will find the following information and write it in your table
  - > write the domain and range
  - > draw the end behavior
  - > graph the function with a (-) coefficient
  - > does the domain & range change
  - > does the end behavior change
- You will have 10 mins to complete this task

If you were able to graph the functions, you can see that the end behaviors for an even degree all look the same...

End Behavior-Even Degree

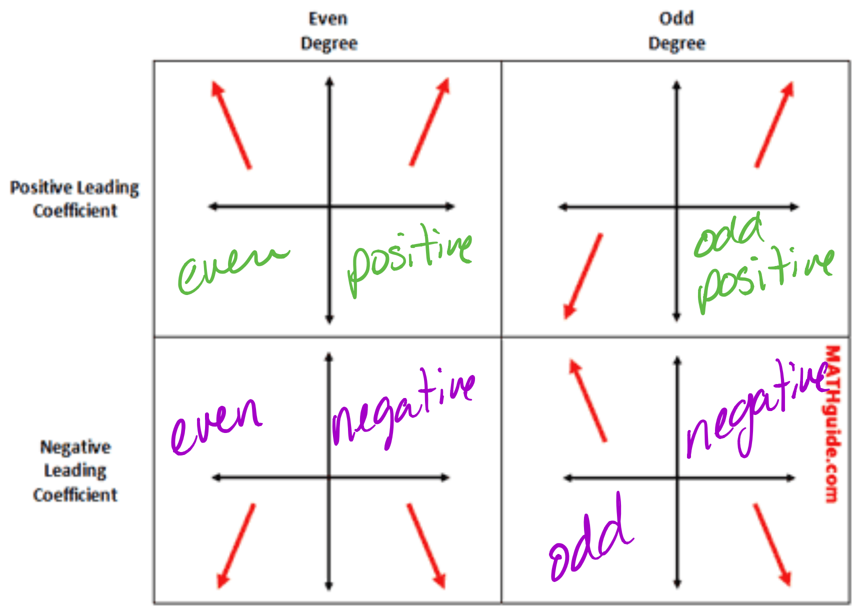
Function	End Behavior left end	End Behavior Right end
$f(x) = x^2$	↑ $x \to -\infty$ $y \to \infty$	↗ $x \to \infty$ $y \to \infty$
$f(x) = x^4$	↖	↗
$f(x) = x^6$	↖	↗

The same goes for odd behaviors.

End Behavior-Odd Degree

Function	End Behavior left end	End Behavior right end
$f(x) = x$	$\swarrow$ $x \to -\infty$ $y \to -\infty$	$\nearrow$ $x \to \infty$ $y \to \infty$
$f(x) = x^3$	$\swarrow$	$\nearrow$
$f(x) = x^9$	$\swarrow$	$\nearrow$

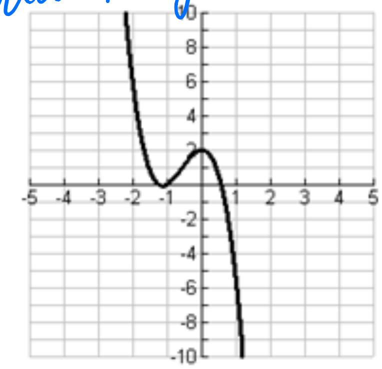
Knowing if the function has a degree that is odd or even and if the function is positive or negative. Tells you the end behavior and can help you graph polynomials



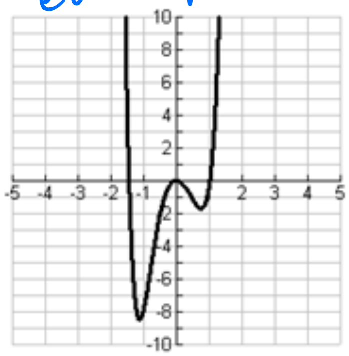
Now let's practice...

Name the degree (even or odd?) & the leading coefficient (positive or negative?

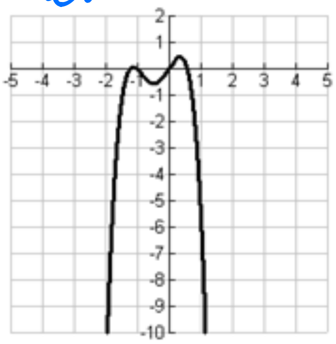
odd negative



even positive



even negative



Since the end behaviors go in opposite directions -one going up -one going down ($x \to -\infty$ $y \to \infty$, $x \to \infty$, $y \to -\infty$)

We know this is odd.

And since odd positive looks like this ↙ ↗ this must be negative because it is flipped ↖ ↘

end behaviors both going in the same direction means even ($x \to -\infty$, $y \to \infty$ $x \to \infty$, $y \to \infty$)

And both up so it's positive

Similar to the middle graph but going down.

So this is even but it is negative

Now let's talk about multiplicity. If I have a function $f(x)=(x-2)^2(x-1)(x+3)^3$ and I solve for the zeros, I set the factors equal to zero ... like this...

$(x-2)^2$: $(x-2)=0$ $(x-2)=0$ $(x-1)=0$

$(x+3)^3$: $(x+3)=0$ $(x+3)=0$ $(x+3)=0$

$x=2$ $x=2$ $x=1$ $x=-3$ $x=-3$ $x=-3$

Multiplicity describes how many times a zero occurs. For example ... the zero $x=2$ occurs 2 times. So it has a multiplicity of 2. The zero $x=-3$ occurs 3 times so it has a multiplicity of 3.

Multiplicity

Multiplicity can help us graph...

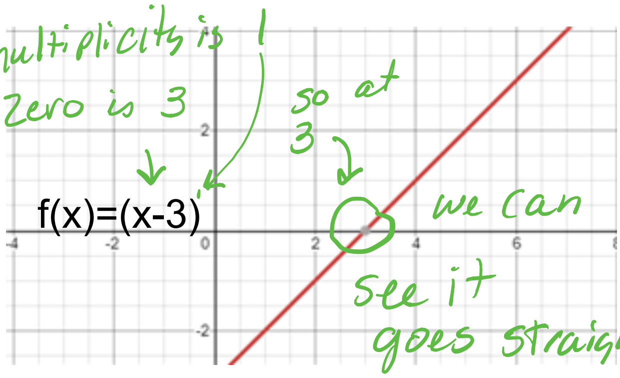
Straight intersection:

If the factor has a multiplicity of 1 the intersection will be straight.

$(x - a)^1$ The power of the zero is 1.

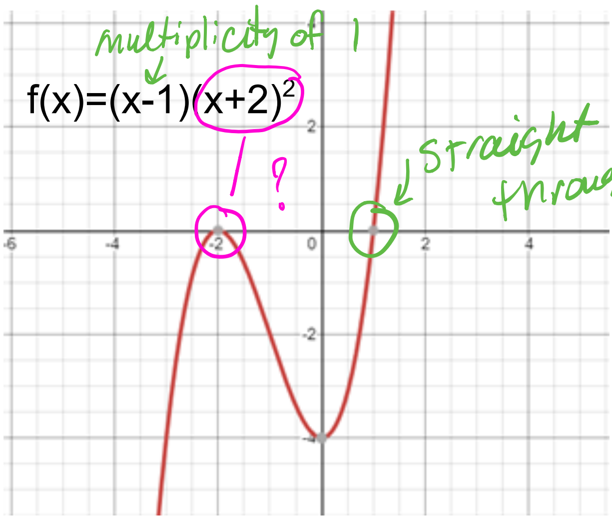
multiplicity is 1

zero is 3



so at 3 we can see it goes straight through the point

multiplicity of 1



? straight through point

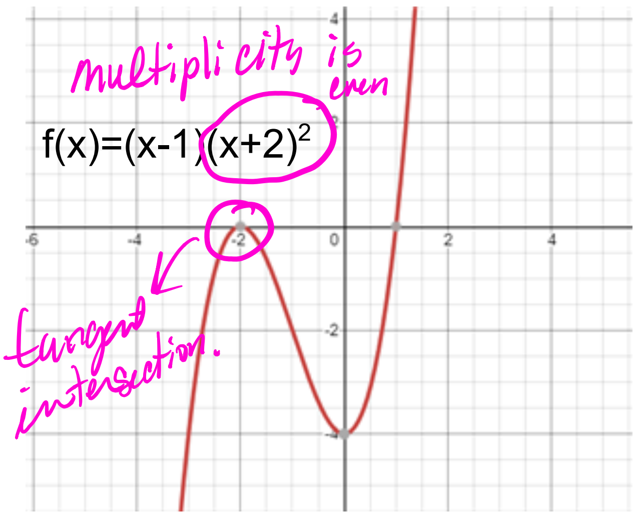
move on to see what a multiplicity of an even # is called.

$(x+2)^2$ has a multiplicity of 2 which is even.
If a factor has a multiplicity of an even number it will have a

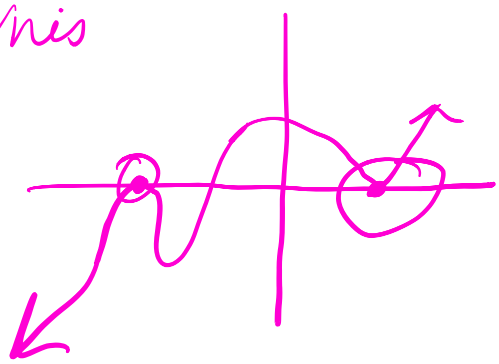
Multiplicity

Tangent intersection : (like a bounce) → means it hits the point and bounce in the opposite direction.

$(x - a)^{even}$ The power of the zero is even.



a bounce looks like this



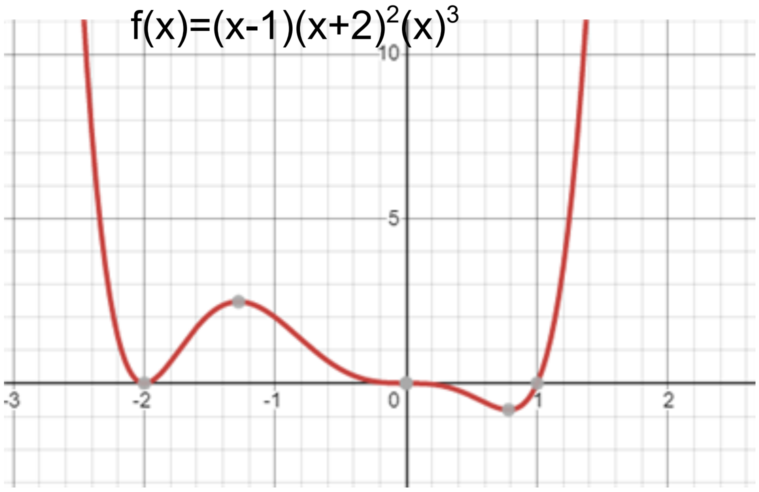
What if the multiplicity is odd...

must be greater than 1.

Multiplicity $(x)^3$ multiplicity is odd so it will be an

Inflection intersection: **(like a wiggle through)** - this is like a slide through.

$(x - a)^{odd}$ The power of the zero is odd and >1.



It slides or wiggles through the point.

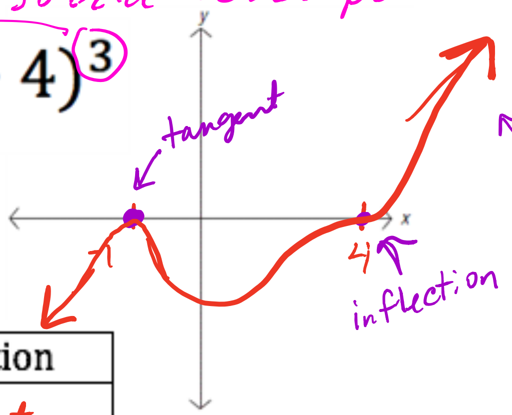
When graphing you want to determine end behavior first. So is the function odd or even and is it positive or negative?

Ex. 8 Find the zeros, the multiplicity, end behavior and graph the following:

5 so odd and positive

$$f(x) = (x+1)^2(x-4)^3$$

then find the zeros, multiplicity & type of Intersection



to graph
1st graph end behaviors
2nd plot the zeros at -1 and 4
3rd draw the type of intersection at the zero of -1 and 4.

Zeros	Multiplicity	Intersection
-1	2	tangent
4	3	inflection

$x+1=0$ $\quad$ $x-4=0$

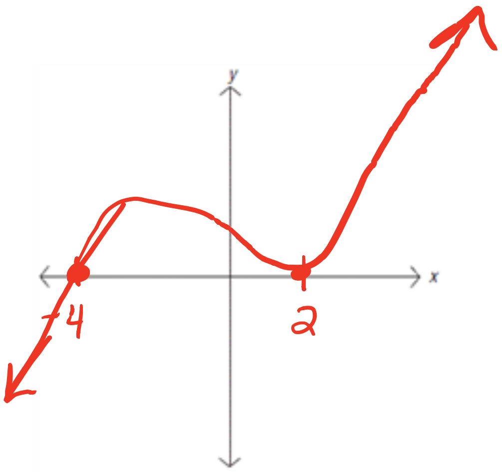
$x=-1$ $\quad$ $x=4$

multiplicity of 2 $\quad$ multiplicity of 3

Let's try it again

$f(x) = (x-2)^8(x+4)$

odd positive ↙ ↗



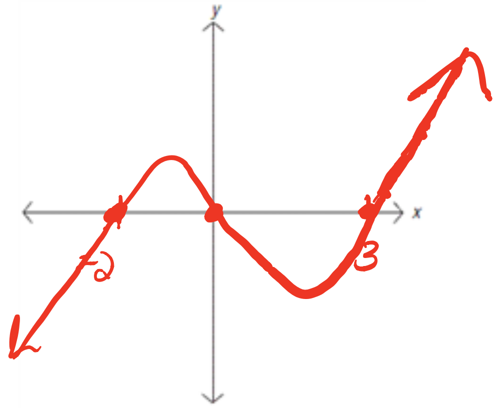
Zeros	Multiplicity	Intersection
2	8	tangent
-4	1	straight.

again

when no power it is (1).

$$f(x) = x^1(x+2)^1(x-3)^1$$

don't forget this zero

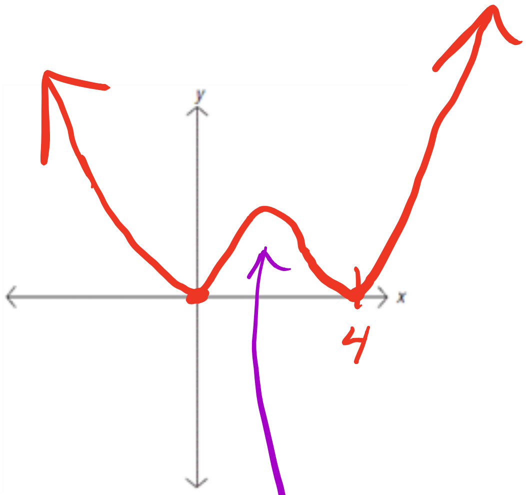


Zeros	Multiplicity	Intersection
0	1	Straight
-2	1	Straight
3	1	Straight

try again

$$f(x) = x^2(x-4)^2$$

even positive

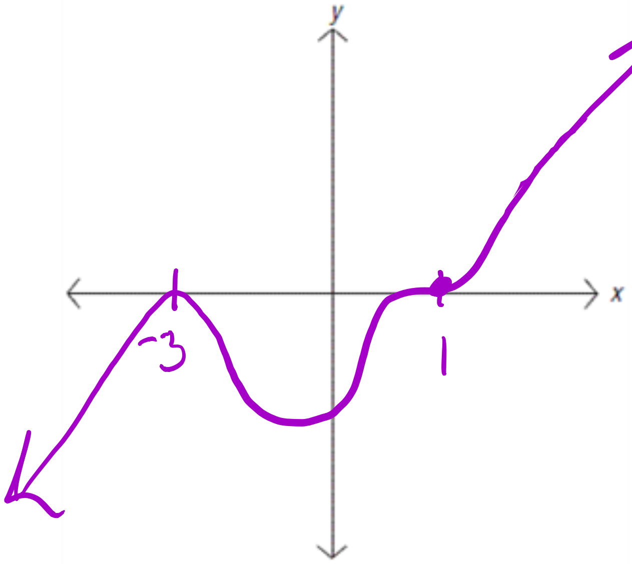


Zeros	Multiplicity	Intersection
0	2	tangent
4	2	tangent

what happen in between the zeros right now is up to you. We will not be getting the detailed.

$$(x+3)^2(x-1)^5=0$$

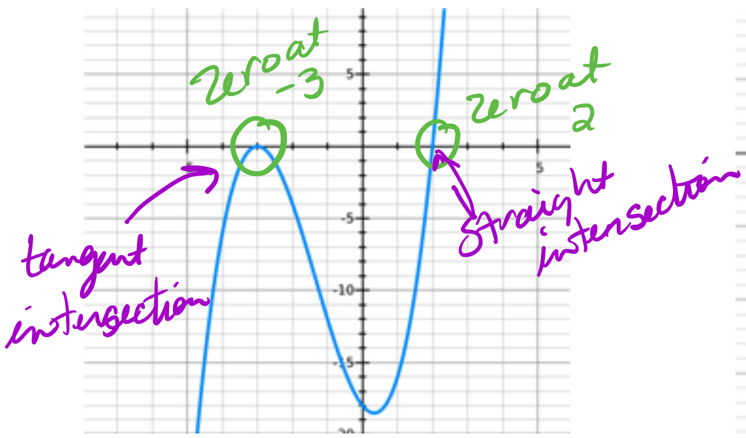
odd positive

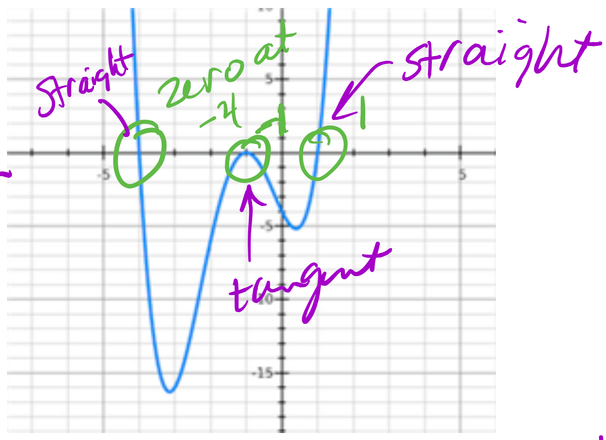


zeros	mult	inters
-3	2	tangent
1	5	inflection

To write an equation from a graph - find the zeros, and write as a factor... determine how they intersect at the zero to determine multiplicity. And then right you function.

Write the equation for the following graphs





if -3 is the zero
$x+3$ is the factor

$$f(x) = (x+3)^2(x-2)^1$$

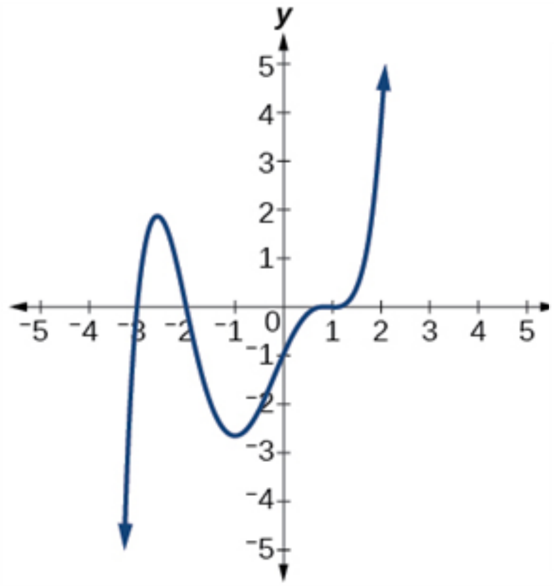
$$f(x) = (x+4)^1(x+1)^2(x-1)^1$$

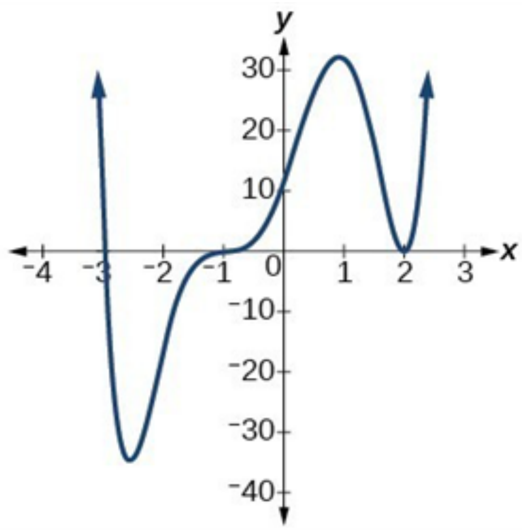
even positive
matches the end behavior.

You can double check your end behaviors.
End behaviors of the graph say is should be odd & positive... and our function that we wrote is odd and positive

Now you try...

Write the equation for the following graphs





Attachments

HW KEY 8-2 Zeros of Polynomials.pdf