8-2 Zeros of a Polynomial

Objectives:

8-2a: I can find the zeroes of a polynomial using factoring.

Vocabulary:

Zeros of a Polynomial - a number used for the variable in a function that causes the function to equal zero. Example $f(x) = 2x^2 - 2$

$0 = 2x^2 - 2$
$+2 \quad +2$
$\frac{2}{2} = \frac{2x^2}{2}$
$\sqrt{x^2} = \pm\sqrt{1}$
$x = 1$
zero

a zero is also where a graph intersects the x-axis or the x-intercepts.

Division, Graph, Factor

Tools to help you find the zeros of a polynomial.

You can use synthetic division, a graph or factor the polynomial. Sometimes you may have to use more than one tool.

If this is a zero what is the factor...

If 2 is a zero the factor is $(x-2)$.

If -2 is a zero the factor is $(x+2)$.

If $2, -2, 0$ are the zeros the factors are $(x-2)(x+2)(x)$.

We will see more like this later. Let's look at it the other way...

If this is a factor what is the zero...

If $(x+2)$ is a factor the zero would be 2

If $(x-2)$ is a factor the zero would be -2

If $(x+2)(x+3)(x-5)$ are factors the zeros would be $-2, -3, 5$

So you think you got it... what about this one...

If the factors are $(2x+1)(3x-5)$ what are the zeros...

To find the zeros we solve for "x" in the factors.

$$2x+1=0$$
$$-1 \quad -1$$
$$\frac{2x}{2} = \frac{-1}{2}$$
$$x = \boxed{-\frac{1}{2}} \text{ our zero}$$

$$3x-5=0$$
$$+5 \quad +5$$
$$\frac{3x}{3} = \frac{5}{3}$$
$$x = \boxed{\frac{5}{3}} \text{ our zero}$$

function written in standard form.

$$f(x) = x^3 - 2x^2 - 8x$$

What is the degree of the polynomial?

How many factors does it have?

How many zeros does it have?

How many x-intercepts does it have?

First let's define what a degree of the polynomial is.

The degree is the largest exponent of the variable.

So in $x^3 - 2x^2 - 8x$ the "largest exponent" is 3.

So $x^3 - 2x^2 - 8x$ has a degree of 3.

The degree tells us many things...

How many factors...

How many zeros...

How many x-intercepts...

not in standard form... so we must add the exponents.

Exponents → 2, 1, 3, 1

$$f(x) = x^2(x-1)^1(x+3)^3(x-7)^1$$

$2+1+3+1 = 6$

What is the degree of the polynomial? (6)

How many factors does it have? (6)

How many zeros does it have? (6)

How many x-intercepts does it have? (6)

The Fundamental Theorem of Algebra state...

You will have "n" roots for the "n^{th}" degree.

So how do we find the zeros of a polynomial... we use our tools of factor, division and/or graph.

Let's start with factoring.

Find the zeros of the polynomial using factoring

$2x^2 + x + 10x + 5$ ← to factor we group

GCF GCF

$x(2x+1) + 5(2x+1)$

$(2x+1)(x+5)$ → these are our factors.

How many factors are there... 2

How many zeros are there? What is the degree?

2 2

To get the zeros we set each factor to zero and solve for x.

$2x + 1 = 0$

$-1 \quad -1$

$\frac{2x}{2} = \frac{-1}{2}$

$x = -\frac{1}{2}$

$x + 5 = 0$

$-5 \quad -5$

$x = -5$

So our zeros are

$x = \frac{}{2}$ $-\frac{1}{2}, -5$

Find the zeros by factoring

$$f(x) = x^3 - 2x^2 - 8x$$

← Start with GCF or Greatest common factor.

GCF $x(x^2 - 2x - 8)$ ← Riddle method

$1 \cdot -8 = \frac{-8}{}$

$-1, 8$

$-8, 1$

$-2, 4$

$-4, 2$ · multiply to be -8 add to be -2

$x(x-4)(x+2)$

Set each factor equal to zero and solve for x.

$x = 0$ Already solved

$x - 4 = 0$

$+4 \quad +4$

$x = 4$

$x + 2 = 0$

$-2 \quad -2$

$x = -2$

Zeros are $0, 4, -2$

Let's try again

Find the zeros by factoring

$$f(x) = 2x^3 + 5x^2 - 3x$$ ← GCF

GCF

$x(2x^2 + 5x - 3)$

$2 \cdot -3 = -6$

$-1, 6$
$-6, 1$
$-2, 3$
$-3, 2$

$x([2x^2 + 6x][-1x - 3])$ ← Rewrite $5x$ as $6x - 1x$ then group

GCF

$x(2x(x+3) - 1(x+3))$

$x(2x-1)(x+3)$

Set each factor to zero & solve.

$x = 0$

$2x - 1 = 0$

$x = \frac{1}{2}$

$x + 3 = 0$

$x = -3$

Zeros $0, \frac{1}{2}, -3$

Find if -4 is a zero of the function.

$$3x^2 + 7x - 20$$

To find if (-4) is a zero of the funtion, plug in (-4) for x and see if...

$$3(-4)^2 + 7(-4) - 20 = 0$$

$$3(16) - 28 - 20 = 0$$

$$48 - 48 = 0 \checkmark$$

Yes -4 is a zero

Given that "3" is a zero, find the remaining zeros.

$$x^3 - 4x^2 + x + 6 \div (x-3)$$

to do this we need to use our division tool.

why are we dividing by $(x-3)$?

$x^3 - 4x^2 + x + 6 \div x - 3$

use the coefficients

find the zero

3

multiply diagonally
add vertically

$$\begin{array}{r|cccc} 3 & 1 & -4 & 1 & 6 \\ & & 3 & -3 & -6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

write with new coeffients.

$(x-3)(x^2 - 1x - 2)$ ← factor this

-2
$-2, 1$

$(x-3)(x-2)(x+1)$ ← all the factors - now find the zeros.

$3, 2, -1$ are the zeros.

Given that "1" is a zero, find the remaining zeros.

$f(x) = x^3 + 5x^2 - x - 5 \div (x-1)$ division

$$\begin{array}{r|rrrr} 1 & 1 & 5 & -1 & -5 \\ & & 1 & 6 & 5 \\ \hline & 1 & 6 & 5 & 0 \end{array}$$

$(x-1)(x^2+6x+5)$ factor

$\frac{5}{1,5}$

$(x-1)(x+5)(x+1)$ find zeros

$1, -5, -1$

Now you try it.

Given that "7" is a zero, find the remaining zeros.

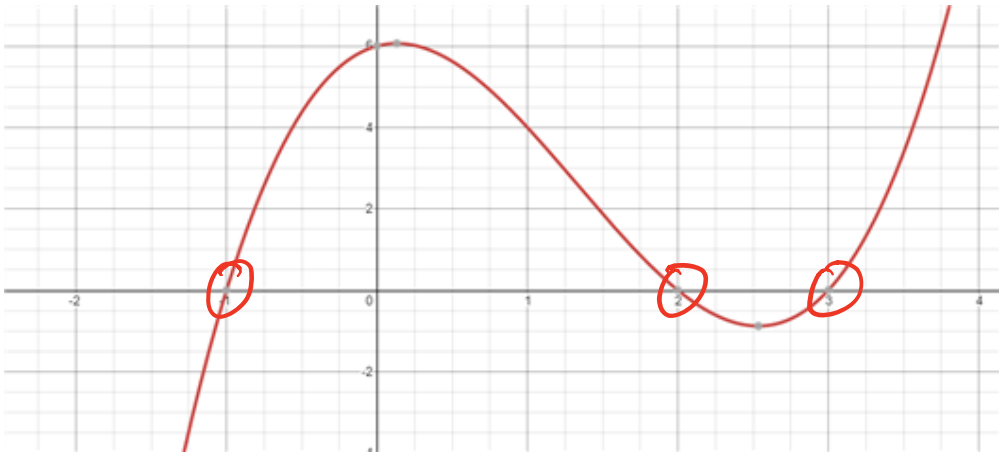
$$f(x) = x^3 - 7x^2 - x + 7$$

If you want to know if you did it correctly... plug your zeros in for "x" in $x^3 - 7x^2 - x + 7$ and see if it equals zero.

Remember a zero is a number for "x" that causes the funtion $f(x) = 0$. And if graphed it shows us the x-intercepts. therfore to...

Find the zeros graphically - we just need to find the x-intercepts.

$$f(x) = x^3 - 4x^2 + x + 6$$



So our zeros are

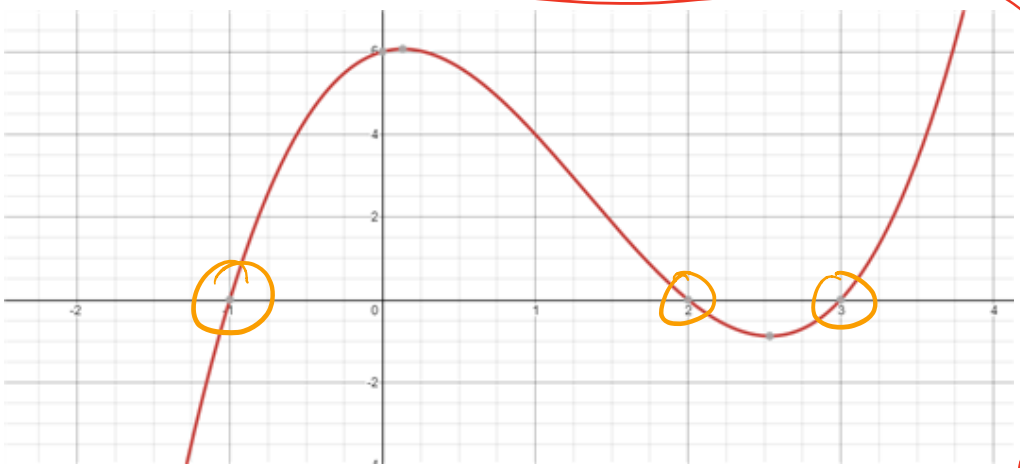
$-1, 2, 3$

Remember x^3 tell us our degree is 3 and so we have 3 zeros, 3 factors, and 3 x-intercepts.

Find the zeros graphically

$$f(x) = (x+1)(x-3)(x-2)$$

factored form of the function above.



notice 3 factors

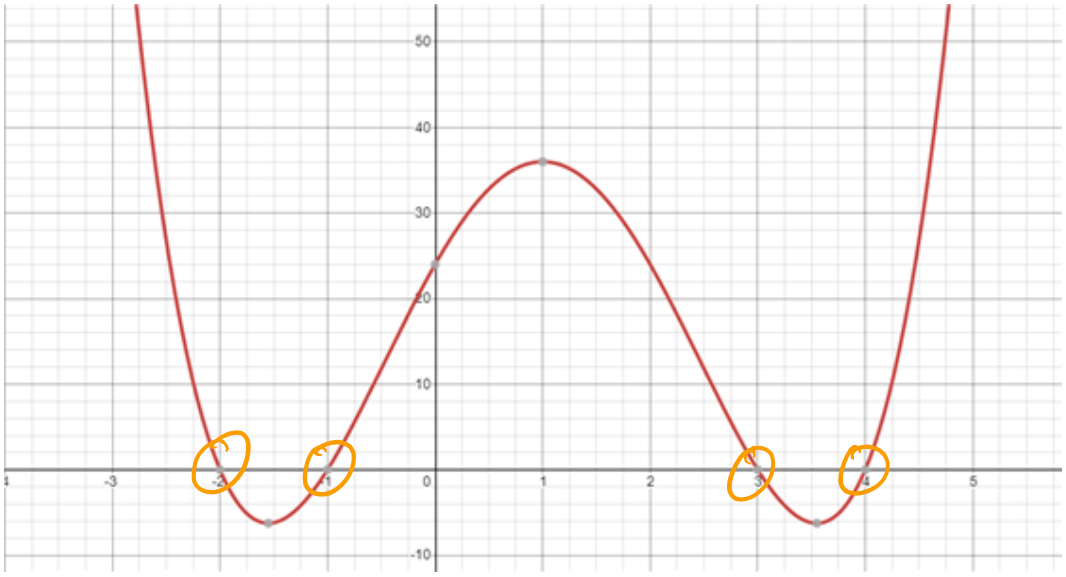
our zeros are

$-1, 2, 3$

How many zeroes will the following have? 4

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

Find them by graphing



$-2, -1, 3, 4$

When you know the zeros you can find the factors and write a polynomial function.

Write a polynomial function in FACTORED form from the given zeros.

$x = 2, -3, 1$

$x = 2 \quad x = -3 \quad x = 1$

$x - 2 = 0 \quad x + 3 = 0 \quad x - 1 = 0$

$(x-2) \quad (x+3) \quad (x-1)$

$f(x) = (x-2)(x+3)(x-1)$

$x = \frac{1}{2}, -4, -\frac{2}{3}$

not doing these steps

$f(x) = (x - \frac{1}{2})(x+4)(x+\frac{2}{3})$

or

$(2x+1)(x+4)(3x+2)$

Now you try it...

Write a polynomial function in FACTORED form from the given zeros.

$x = 0, -5, 2$

To double check take your 3 factors and set them equal to zero and solve for "x" to see if you get 0, -5, 2.

Attachments

HW KEY 8-1 Polynomial Division.pdf