8-1 Dividing Polynomials

Objectives:

****8-1a: I can divide polynomials using synthetic division.****

8-1b: I can determine if a polynomial is a factor of another using division.

Vocabulary

$(3x^4 - 20x^3 + 27x^2 - 9x + 3) \div (x - 5)$

Coefficients - the number multiplied by the variable

divisor - the number dividing into another number. A number in which another number is to be divided

dividend - a number to be divided by another number.

quotient - the answer to a division problem.

Zeros - any number used for the variable that produces an answer of zero. A zero is also where the graph crosses the X-axis also know as the X-intercepts.

factors - a number when multiplied with another

number produces a given number.

Synthetic Division (only with linear divisor)

1) Find the "zero" of the divisor. $(x-5)$ zero $x-5=0$ $+5\ +5$ $x=5$ zero

2) Identify the coefficients of the dividend. $3x^4-20x^3+27x^2-9x+3$

3) Set up 3 rows for synthetic division. $3\ -20\ 27\ -9\ 3$

4) Add vertically and multiply diagonally.

5) Write quotient using new coefficients.

$$(3x^4 - 20x^3 + 27x^2 - 9x + 3) \div (x - 5)$$

5	3	-20	27	-9	3
		15	-25	10	5
	3	-5	2	1	8

remainder and is expressed as

these are the new coefficients - but we start with x^3. (one down from the original expression)

$$3x^3 - 5x^2 + 2x + 1 + \frac{8}{x-5}$$

Is $(x-5)$ a factor of

$3x^4 - 20x^3 + 27x^2 - 9x + 3$!

why or why not?

Let's try again

$$(x^3 + 3x^2 - 4x - 12) \div (x + 3)$$

$x + 3 = 0$ $\quad x = -3$

-3 | 1 3 −4 −12

−3 0 +12

1 0 −4 0 ← no remainder so $(x+3)$ is a factor.

$1x^2 + 0x - 4 + \frac{0}{x+3}$

$(x^2 - 4)$

Let's make sure we did it correctly

$(x+3)(x^2-4) = x^3-4x+3x^2-12$

$x^3+3x^2-4x-12$

Let's try again

$(2x^5 - 14x^4 + 5x^3 + 6x^2 - 5x + 16) \div (x - 1)$

$x-1=0$
$+1 \quad +1$
$x=1$

$$\begin{array}{r|cccccc} 1 & 2 & -14 & 5 & 6 & -5 & 16 \\ & & 2 & -12 & -7 & -1 & -6 \\ \hline & 2 & -12 & -7 & -1 & -6 & \boxed{10} \end{array}$$

10 - remainder

$2x^4 - 12x^3 - 7x^2 - 1x - 6 + \frac{10}{x-1}$

(A) $(7x^3 - 6x + 9) \div (x + 5)$

$x + 5 = 0$
$-5 \quad -5$
$x = -5$

notice there is no x^2

But when doing synthetic division we must put in a place holder for any missing variable with the "next" power down. So...

$$\begin{array}{r|rrrr} -5 & 7 & 0 & -6 & 9 \\ & & -35 & 175 & -845 \\ \hline & 7 & -35 & 169 & -836 \end{array}$$

-836 remainder

$$7x^2 - 35x + 169 - \frac{836}{x+5}$$

So $x+5$ is Not a factor of $(7x^3 - 6x + 9)$.

What does it mean for a number to be a factor of another? Think of 9 and 36... is 9 a factor of 36.

How do you tell if a number is a factor?

$36 \div 9 = 4$

$9 \times 4 = 36$

Factors factors

Using this same concept - let's try it with polynomials...

Determine if $d(x)$ is a factor of $f(x)$.

2) $f(x) = 4x^2 - 18x + 8$
$d(x) = x - 4$ $\quad x - 4 = 0$, $+4 \ +4$, $x = 4$

To determine if $d(x)$ is a factor of $f(x)$, we must divide $f(x)$ by $d(x)$. If it divides evenly then it is a factor. Use synthetic division

$$\begin{array}{r|rrr} 4 & 4 & -18 & 8 \\ & & 16 & -8 \\ \hline & 4 & -2 & 0 \end{array}$$

no Remainder so it divides it evenly - therefore it is a factor.

$(4x-2)(x-4)$

We can double check this by multiplying our factors.

$(4x-2)(x-4) = 4x^2 - 16x - 2x + 8$

$4x^2 - 18x + 8 = f(x)$ ✓

Determine if $d(x)$ is a factor of $f(x)$.

2) $f(x) = 5x^5 - 30x^4 + x^3 - 7x^2 + 8x$
$d(x) = x - 6$ $x-6=0$ $+6\ +6$ $x=6$

6	5	-30	1	-7	8	0
		30	0	6	-6	12
	5	0	1	-1	2	(12)

remainder

so $d(x)$ is Not a factor of $f(x)$