5-3 Solving Exponential and Logarithmic equations

Objectives:

5-3a: I can solve exponential and logarithmic equations graphically.

5-3b: I can solve exponential and logarithmic equations algebraically.

RECALL: Solving Graphically

$$275e^{0.06x} = 1000$$

$y_1 = 275e^{0.06x}$ $y_2 = 1000$

$$10^{2x} = 1500$$

$y_1 = 10^{2x}$ $y_2 = 1500$

Now you try...solve the exponential & logarithmic equations graphically.

$20^{2r} = 56$

$y_1 = 20^{2r}$ $y_2 = 56$

$e^{.23x} = 1.99$

$y_1 = e^{.23x}$ $y_2 = 1.99$

Inverses - what do I do to undo the operation and slove for X?

Addition/Subtraction	Natural Log/e^	Common Log/10^	Log base b/b^
$x-5=10$ $+5 \quad +5$ $X=15$ So we add / Subtraction	$e^x=5$ Take the ln of both sides... $\ln e^x=\ln 5$ $x=\ln 5$	$10^x=100$ $\log_{10}100=X$	$2^x=16$ $\log_2 16=X$
$x+7=21$ $-7 \quad -7$ $X=14$ Addition / So we Subtract	$\ln x=7$ write as an exponent... $e^7=X$	$\log x=3$ $10^3=X$	$\log_3 x=4$ $3^4=X$

Remember ln is $\log_e$

To Solve exponential and logarithm equations...
- 1st you must get the base with the variable exponent isolated by using inverse operations.

Solve the following equations

$$\frac{10}{5} = \frac{5e^{4x}}{5}$$

$2 = e^{4x}$ ← To isolate the e^{4x} divide by 5.

Next take the natural log (ln)

$\ln 2 = \ln e^{4x}$

$\frac{\ln 2}{4} = \frac{4x}{4}$

$x = \frac{\ln 2}{4}$

$$\log_3(2x-4) = 4$$

$3^4 = 2x - 4$

$81 = 2x - 4$
$+4 \quad +4$

$\frac{85}{2} = \frac{2x}{2}$

$\boxed{42.5 = x}$

$$5^{x-1} - 4 = 7$$
$+4 \quad +4$ ← add 4 to both sides.

$5^{x-1} = 11$

$\log_5 11 = x - 1$
$+1 \quad +1$

$\log_5 11 + 1 = x$

$$6^{3x} = 12$$

$\frac{\log_6 12}{3} = \frac{3x}{3}$

$x = \frac{\log_6 12}{3}$

$3\ln 2$ can also be written as $\ln 2^3 = \ln 8$

$e^{\ln 8} = 8$

Solve the following

$\ln(x+12) = 3\ln 2$

$e^{3\ln 2} = x+12$ *

$*\ e^{\ln 8} = x+12$

$8 = x+12$

$-12 \quad -12$

$-4 = x$

$\log(4x) = 2$

$10^2 = 4x$

$\frac{100}{4} = \frac{4x}{4}$

$x = 25$

$4\ln(x+7) - 5 = 1$

$+5 \quad +5$

$\frac{4\ln(x+7)}{4} = \frac{6}{4}$

$\ln(x+7) = \frac{3}{2}$

$e^{3/2} = x+7$

$-7 \quad -7$

$e^{3/2} - 7 = x$

$-2.52 = x$

Solve the following

$\log(x+2)-3=5$

$+3 \quad +3$

$\log(x+2)=8$

$10^8=x+2$

$-2 \quad -2$

$10^8-2=x$

$99,999,998=x$

$\log_4(x-1)=1$

$4^1=x-1$

$+1 \quad +1$

$5=x$

Remember this from unit 4?...

Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

P is the principal

r is the annual interest rate

n is the number of compounding periods per year

t is the time in years

Many banks compound the interest on accounts daily or monthly. However, some banks compound interest continuously, or at every instant, by using the *continuous compounding formula.*

And this?... **Continuous Compounding Formula**

If *P* dollars are invested at an interest rate *r*, that is compounded continuously, then the amount, *A*, of the investment at time *t* is given by

$$A(t) = Pe^{rt}$$

P is principal (initial value)

r is interest rate

t is time (in years)

Before we would graph to solve... but now we have the skills to solve algebraically.

First determine which equation to use and then set it up.

How long will it take for a $250 initial investment in an account that pays 4.5% compounded continuously to grow to $750?

$250 = P$
$750 = A(t)$
$4.5 = .045 = r$
t = unknown

$\frac{750}{250} = \frac{250(e)^{.045t}}{250}$ next undo to isolate $e^{.045t}$

$3 = e^{.045t}$ take the natural log of both sides

$\ln 3 = \ln e^{.045t}$ $\quad \frac{\ln 3}{.045} = \frac{.045t}{.045}$ $\quad \frac{\ln 3}{.045} = t$ $\quad t = 24.4$ years

Suppose that $250 is deposited into an account that pays 4.5% compounded quarterly. Solve for t to find how long it will take for the account to contain at least $500.

$P = 250$
$4.5 = .045 = r$
$n = 4$ (quarterly)
$A(t) = 500$
t = unknown

$\frac{500}{250} = \frac{250\left(1 + \frac{.045}{4}\right)^{4t}}{250}$

$2 = (1.01125)^{4t}$ - write as a log

$\frac{\log_{1.01125} 2}{4} = \frac{4t}{4}$

$\frac{\log_{1.01125} 2}{4} = t$

Comparing acidity: $pH = -\log\left[H^+\right]$ ← use this to help solve

$\left[H^+\right]$ = hydrogen-ion concentration

Vinegar has a pH of 2.4. What is it's hydrogen ion concentration?

$\frac{2.4}{-1} = \frac{-\log[H^+]}{-1}$ $-2.4 = \log[H^+]$ $(10)^{-2.4} = [H^+]$ $[H^+] = .004$

Baking soda has a pH of 8.4. What is it's hydrogen ion concentration?

$\frac{8.4}{-1} = \frac{-\log[H^+]}{-1}$ $(10)^{-8.4} = [H^+]$

$-8.4 = \log[H^+]$ $[H^+] = .000000004$

Which has a higher hydrogen ion concentration?

Vinegar has a higher hydrogen ion concentrate

Home work 5.3