Starter: Evaluate the expression **WITHOUT** using a calculator.

1). $\log_4 64$

$\log_4 4^{3} = 3$

2). $\log_6 3 + \log_6 12$

We will talk about this later...

Rewrite the equation in logarithmic form.

3). $4^x = 16$

$\log_4 16 = x$

4). $6^2 = y$

$\log_6 y = 2$

QUIZ... 5.1

Clear your desks...

NO Calculator Needed

Exponent Rules Review

$2^5 \cdot 2^3 = 2^{5+3} = 2^8$

$\frac{2^5}{2^3} = 2^{5-3} = 2^2$

$(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$

$\sqrt[3]{8} = 8^{1/3}$

$2^{-1} = \frac{1}{2}$

5-2: Properties of Logarithms

5-2a: I can use the properties of logarithms to simplify and evaluate logarithms.

Find the value of each logarithm without using a calculator.

1. $\log_7 7$ $\quad 7^x = 7^1 \quad x = 1$

2. $\log_{18} 18 = 1$ $\quad 18^x = 18^1 \quad x = 1$

3. $\log_5 1 = 0$ $\quad 5^x = 1 \quad 5^0 = 1 \quad x = 0$

4. $\log_9 1 = 0$ $\quad 9^x = 1 \quad 9^0 = 1 \quad x = 0$

$$\log_a 1 = 0 \qquad \log_a a = 1$$

follow Rule

$\log_5 1$ = 0

$\log_4 4$ = 1

$\ln 1$ = 0

Remember ln is saying natural log or Log base e.

$\log 10$ = 1

a log with no base is just log base 10.

Evaluate the logarithm:

- $\log_3 3^2$ $3^x = 3^2$ so $x = 2$
- $\log_5 5^8$ $5^x = 5^8$ so $x = 8$

Without evaluating, predict what the following logs equal:

- $\log_2 2^{10}$ $= 10$
- $\log_{20} 20^7$ $= 7$

Inverse Property of Logarithms
If b and r are positive real numbers, with $b \neq 0$, then

$$\log_a a^r = r$$

this says if you have $\log_a a^r$ it will equal the exponent r.

$\log_4 4^{3} = 3$

$\ln e^{-0.5} = -.5$

Recall: $b^x = a \iff \log_b a = x$

How would we write the following exponential as a log?

$$5^{\log_5 20} = X$$

I know this looks scary - but don't be afaid. Let's identify what we know...

The base is 5. The exponent is $\log_5 20$. And we are not give what it equals so we will say it is X.

Now write it as a log...

$\log_5 X = \log_5 20$ so...

$x = 20$

The property is next...

Inverse Property of Logarithms

If b and M are positive real numbers, with $b \neq 0$, then

$\log_{20} 20^7 = 7$

$$b^{\log_b M} = M$$

$5^{\log_5 20} = 20$

$8^{\log_8 \sqrt{23}} = \sqrt{23}$

$12^{\log_{12} \sqrt{2}} = \sqrt{2}$

$10^{\log 0.2} = 0.2$

Product Rule of Logarithms
If M N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(MN) = \log_b M + \log_b N$$

Which exponent rule is this similar to?

$x^2 \cdot x^3 = x^{2+3}$

Why would we want to be able to split up a logarithm?

To help use solve more easily.

Write each of the following logarithms as the sum of logarithms.

$\log_2(5 \cdot 3)$ $\qquad$ $\ln(6z)$

$\log_2 5 + \log_2 3$ $\qquad$ $\ln 6 + \ln z$

Find 3 ways to expand $\log_3 24$ using this rule

$\log_3 3 + \log_3 8$ $(3 \cdot 8 = 24)$

$\log_3 6 + \log_3 4$ $(6 \cdot 4 = 24)$

$\log_3 12 + \log_3 2$ $(2 \cdot 12 = 24)$

$\log_3 1 + \log_3 24$ $(24 \cdot 1 = 24)$

How do you predict we would write the following logarithm as two logarithms?

division so we subtract

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

Quotient Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$\log_2\left(\frac{5}{3}\right)$ $\log_2 5 - \log_2 3$ $\log\left(\frac{y}{5}\right)$ $\log y - \log 5$

Find 3 ways to expand $\log_5 3$ using this rule

$\log_5 3 - \log_5 1$ $(3 \div 1 = 3)$

$\log_5 24 - \log_5 3$ $(24 \div 8 = 3)$

$\log_5 33 - \log_5 11$ $(33 \div 11 = 3)$

Write the following as the sum or difference of logarithms.

division

$\log_3\left(\frac{4x}{y}\right) = \log_3 4x - \log_3 y$

$\log_3 4 + \log_3 x - \log_3 y$

but also notice $4x$ is a multiplication problem.

$\log_3\left(\frac{3m}{n}\right) = \log_3 3m - \log_3 n$

$\log_3 3 - \log_3 m - \log_3 n$

$\log_3\left(\frac{q}{3p}\right) = \log_3 q - \log_3 3p$

$\log_3 q - \log_3 3 + \log_3 p$

Show that the following logs are equal:

$4^3 = 4 \cdot 4 \cdot 4$

16

64

$$\log_2(4)^3 = 3 \cdot \log_2 4$$

$\log_2 64 = 3 \cdot 2$

$6 = 6$ ✓

Yes they are the same.

Power Rule of Logarithms
If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r \log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$\log_8 3^5$

$5 \log_8 3$

$\ln x^{\sqrt{3}}$

$\log_{10}$

$\log_5 25 = 2$

$\log b^5$

$10^y = b^5$

These will be posted in the room.

$$\log_a 1 = 0 \qquad \log_a a = 1$$

Power Rule of Logarithms
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If M, N and b are positive real numbers, with $b \neq 0$, then

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If b and M are positive real numbers, with $b \neq 0$, then

$\log_{20} 20^7$

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Product Rule of Logarithms
If M N and b are positive real numbers, with $b \neq 0$, then

$$\log_b (MN) = \log_b M + \log_b N$$

Expand the logarithm.

$\log_2(x^2y^3)$

$\log_2 x^2 + \log_2 y^3$

$2\log_2 x + 3\log_2 y$

$\log\left(\frac{100x}{\sqrt{y}}\right)$

$\log 100x - \log\sqrt{y}$

$\log 100 + \log x - \frac{1}{2}\log y$

$\log_5 \_\_ + \log_5 \_\_ = \log_5 30$

$\log_5 10 + \log_5 3 = \log_5 30$

Write each of the following as a single logarithm.

$\log_6 3 + \log_6 12$ $\quad$ $\log_6 36$

$\log(x-2) - \log x$ $\quad$ $\log \dfrac{(x-2)}{x}$

$\log_5 x - 3\log_5 2$ $\quad$ $\log_5\left(\dfrac{x}{2^3}\right)$

$\log(x-1) + \log(x+1) - 3\log x$

$\log \dfrac{(x-1)(x+1)}{x^3}$

Homework 5.2