4-2 Exponential Equations

4-2a: I can use exponential formulas to model and solve situations of growth and decay.

EXPONENTIAL FUNCTION

$$f(x) = a(b)^x$$

Exponent

Initial Value
(y-intercept)

Base
(Multiplier)

Graph the following functions on a calculator and sketch.

a. $f(x) = 2^x$

b. $f(x) = \left(\frac{1}{2}\right)^x$

What did you notice about the graphs and their equations?

When b>1, the function represents **exponential** ________

When 0<b<1, the function represents **exponential** _______

Exponential Growth and Decay

$$f(t) = a(1 \pm r)^t$$

f(t) = value of the function after time (t)

a = initial value

r = interest rate (written in decimal form)

t = time (in years unless otherwise stated)

John researches a baseball card and find that it is currently worth $3.25. However, it is supposed to increase in value 11% per year. $f(t) = a(1 \pm r)^t$

a) Write an exponential equation to represent this situation

b) How much will the card be worth in 10 years?

c) Use your graphing calculator to determine in how many years will the card be worth $26.

On federal income tax returns, self employed people can depreciate the value of business equipment. Suppose a computer valued at $2765 depreciates at a rate of 30% per year. $f(t) = a(1 \pm r)^t$

a) Write an exponential equation to model this situation

b) How much will this computer be worth in 5 years?

c) Use your graphing calculator to determine in how many years will the computer be worth $350.

The population of Orem in 1950 was 4,000 and was increasing at a rate of 2.6% per year.

a) Predict the population of Orem in 1975 and 2000.

b) Using your graphing calculator, predict when Orem's population will hit 200,000 people.

You try! 🙂

As a birthday present you received a pair of track shoes signed by Mr. Myrup that is valued at $500 (ya know cause he's so awesome). Over time the value increases at a rate 5.5% per year.

a) Write an exponential equation to represent the situation.

b) How much will the shoes be worth after 7 years?

c) How long until they are worth $1000?

Compound Interest Formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

A(t) is the value after time (t)

P is the principal

r is the annual interest rate

n is the number of compounding periods per year

t is the time in years

Write an equation then find the final amount for each investment.

a. $1000 at 8% compounded semiannually for 15 years

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

You Try!

b. $1750 at 3.65% compounded daily for 10 years

Using a calculator, determine how many years it will take for the amount to reach $4000.

Write an equation then find the final amount for each investment.

a) An investment of $1000 comounded monthly at a rate of 4.5%.

b) How much money is there after 5 years?

c) How long until the investment has tripled its value?

Investigate the growth of $1 investment that earns 100% annual interest (r=1) over 1 year as the number of compounding periods, n, increases. Do this with a group/partner.

Compounding schedule	n	$1\left(1+\frac{1}{n}\right)^n$	Value of A
annually	1		
semiannually	2		
quarterly	4		
monthly	12		
daily	365		
hourly	8760		
every minute	525600		

What does the value of A approach?

The value e is called the natural base

The exponential function with base e, $f(x)=e^x$, is called the natural exponential function.

$$e \approx 2.71828182827$$

what you need to know is $e \approx 2.7$

Evaluate $f(x) = e^x$ for

a. $x = 2$

b. $x = \frac{1}{2}$

c. $x = -1$

Many banks compound the interest on accounts daily or monthly. However, some banks compound interest continuously, or at every instant, by using the *continuous compounding formula.*

Continuous Compounding Formula

If P dollars are invested at an interest rate r, that is compounded continuously, then the amount, A, of the investment at time t is given by

$$A(t) = Pe^{rt}$$

A person invests $1550 in an account that earns 4% annual interest compounded continuously.

a. Write an equation to represent this situation

b. Using a calculator, find when the value of the investment reaches $2000.

An investment of $1000 earns an annual interest rate of 7.6%.

Compare the final amounts after 8 years for interest *compounded quarterly* and for interest *compounded continuously.*